

## CONGRUENT TRIANGLES

Congruent triangles have corresponding sides and angles that are congruent. This correspondence between congruent sides and congruent angles is shown in what is called a congruence statement. An example of a congruence statement is

$$\triangle GRA \cong \triangle CEM$$

which tells us that the two triangles are congruent using the notation  $\cong$ . Notice that  $\triangle GRA$  is the same triangle as  $\triangle GAR$  or  $\triangle RAG$ , but the order by which we use the vertices in a congruence statement indicates the correspondence between sides and angles that are congruent. Thus, in this pair of triangles,

$$\begin{aligned}\angle G &\cong \angle C \\ \angle R &\cong \angle E \text{ and} \\ \angle A &\cong \angle M\end{aligned}$$

and that

$$\begin{aligned}\overline{GR} &\cong \overline{CE} \\ \overline{RA} &\cong \overline{EM} \text{ and} \\ \overline{GA} &\cong \overline{CM}.\end{aligned}$$

In your own words: *How is a congruence statement between two triangles formed?*

Try!

A. Given the pairs of congruent angles and congruent sides, write a congruence statement between the two triangles.

1.  $\angle A \cong \angle B$ ,  $\angle C \cong \angle D$ ,  $\angle E \cong \angle F$  and  $\overline{AC} \cong \overline{BD}$ ,  $\overline{CE} \cong \overline{DF}$ , and  $\overline{AE} \cong \overline{BF}$   
Congruence Statement : \_\_\_\_\_

2. If in a triangle,  $\angle M \cong \angle R$ ,  $\angle A \cong \angle N$  and  $\angle X \cong \angle Y$ , and their corresponding sides are congruent, then  $\triangle YAM \cong$  \_\_\_\_\_.

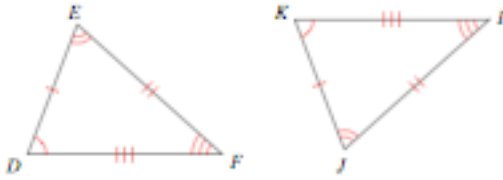
3. If in a triangle,  $\overline{PQ} \cong \overline{RS}$ ,  $\overline{QT} \cong \overline{SU}$  and  $\overline{PT} \cong \overline{QU}$ , and their corresponding angles are congruent, then  $\triangle TPQ \cong$  \_\_\_\_\_.

- B. 1. If  $\triangle JOY \cong \triangle SAD$ , give the pairs of congruent angles and congruent triangles.

2. Give another true congruence statement if  $\triangle NEW \cong \triangle OLD$ .

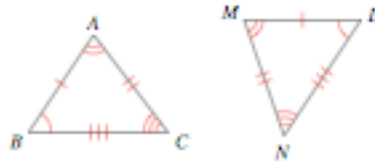
Complete each congruence statement by naming the corresponding angle or side.

1)  $\triangle DEF \cong \triangle KJI$



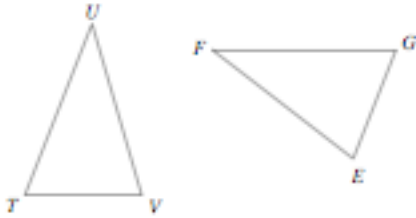
$\overline{FD} \cong ?$

2)  $\triangle BAC \cong \triangle LMN$



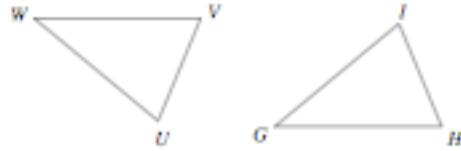
$\angle A \cong ?$

3)  $\triangle TUV \cong \triangle GFE$



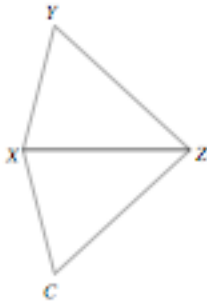
$\angle U \cong ?$

4)  $\triangle WVU \cong \triangle GHI$



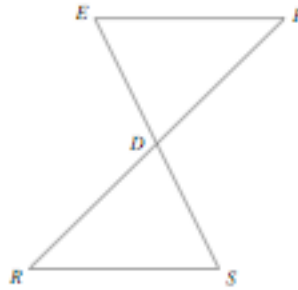
$\angle W \cong ?$

5)  $\triangle ZXY \cong \triangle ZXC$



$\angle Y \cong ?$

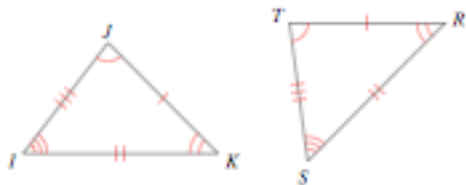
6)  $\triangle DEF \cong \triangle DSR$



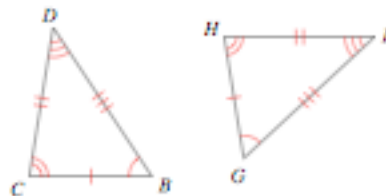
$\angle F \cong ?$

Write a statement that indicates that the triangles in each pair are congruent.

7)



8)



## CONGRUENCE POSTULATES

A **postulate** is a statement that is accepted as true in geometry, even without proof. Congruence postulates allow us to show that two triangles are congruent even without having to show that all six pairs of angles and sides are each congruent.

The following are postulates about congruence between two triangles:

SSS Congruence Postulate  
ASA Congruence Postulate  
SAS Congruence Postulate

### The SSS Congruence Postulate

If each of the three sides of a triangle are congruent respectively to corresponding sides of another triangle, then the two triangles are congruent.

State the SSS Congruence Postulate using  $\triangle ABC$  and  $\triangle DEF$ .

If \_\_\_\_\_, \_\_\_\_\_ and \_\_\_\_\_, then \_\_\_\_\_.

### The ASA Congruence Postulate

If two angles and the included side of one triangle are congruent to two angles and the included side of another triangle, then the two triangles are congruent.

An included side of two angles is the side that is common to the two angles.

State the ASA Congruence Postulate using  $\triangle ABC$  and  $\triangle DEF$ .

If \_\_\_\_\_, \_\_\_\_\_ and \_\_\_\_\_, then \_\_\_\_\_.

### The SAS Congruence Postulate

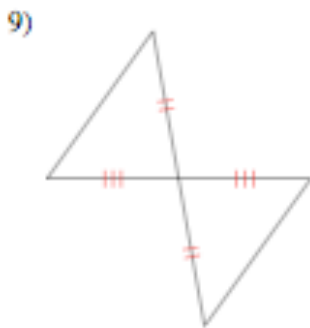
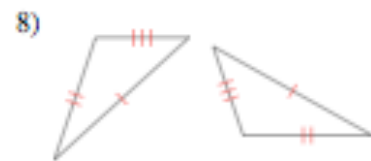
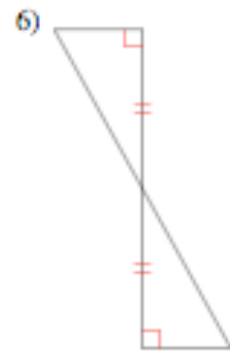
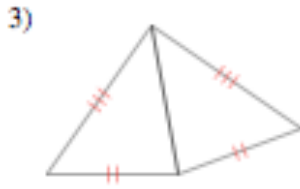
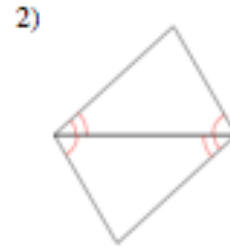
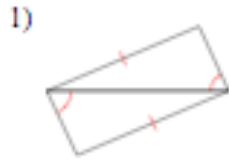
If two sides and the included angle of a triangle are congruent to two sides and the included angle of another triangle, then the two triangles are congruent.

An included angle of two sides is the angle formed by the two sides.

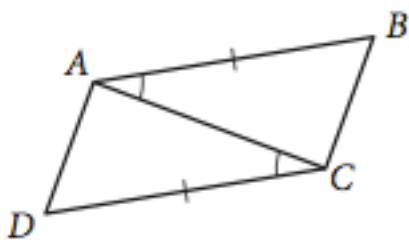
State the SAS Congruence Postulate using  $\triangle ABC$  and  $\triangle DEF$ .

If \_\_\_\_\_, \_\_\_\_\_ and \_\_\_\_\_, then \_\_\_\_\_.

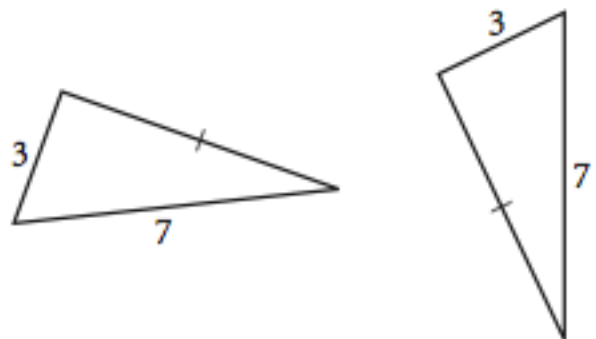
Decide whether the triangles are congruent. If they are, name the congruence shortcut you used to make that conclusion.



11.

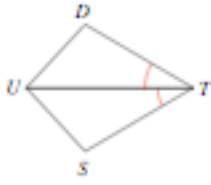


12.

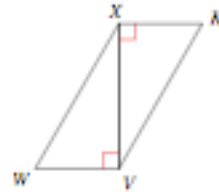


State what additional information is required in order to know that the triangles are congruent for the reason given.

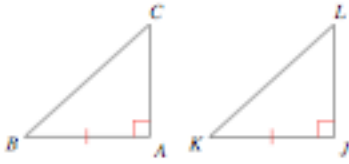
11) ASA



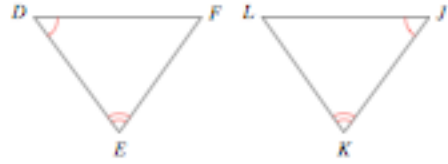
12) SAS



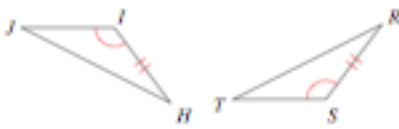
13) SAS



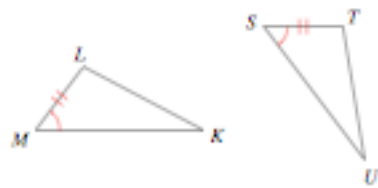
14) ASA



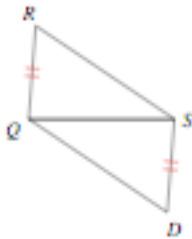
15) SAS



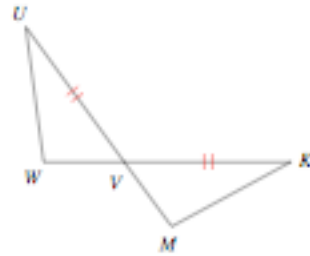
16) ASA



17) SSS



18) SAS



Why can we not conclude yet that two triangles are congruent when two pairs of corresponding angles are congruent? Illustrate.