## Set Relations

## Objectives:

After reading and completing this module, you will be able to do these:
$\checkmark$ Compare sets, using the terms: a) equal and equivalent, b) joint and disjoint.
$\checkmark$ Determine the subsets of a given set.

## LESSON PROPER

## EQUAL SETS

- Two sets $A$ and $B$ are equal, written as $A=B$, if and only if they contain exactly the same elements.


## Example:

Consider the sets

$$
X=\{a, m, y\} \quad Y=\{y, m, a\} \quad Z=\{1 . a . y\} \quad W=\{m a, y\}
$$

$X$ has 3 elements namely $a, m, y$ in alphabetical order.
Y has 3 elements namely a , m , y in reverse alphabetical order.
$X$ and $Y$ are equal sets.
Elements in both sets need not be of the same order.
$X \neq Z$, why?
Because there is an element in X that is not present in Z , and vice-versa.
( $m$ is not an element of $Z$ and $I$ is not an element of $X$ )
$\mathrm{Y} \neq \mathrm{W}$, why?
Because $Y$ has 3 elements namely $y$, $m$, a while $W$ has 2 elements namely ma and $y$.

## EQUIVALENT SETS

- Two sets $A$ and $B$ are equivalent, written $A \sim B$, if and only if they have the same cardinal number;
that is $n(A)=n(B)$.


## Example:

$L=\{$ Luzon, Visayas, Mindanao $\}$
$M=\{r e d$, white, blue $\}$
I = \{red, blue, white $\}$
$n(L)=3, n(M)=3$ and $n(I)=3$.
Since sets $L$, $M$ and $I$ have the same cardinal number which is 3 , then, sets $L, M$, and $I$ are equivalent.

Are there equal sets in the given example above? (Yes, $M=I$ )

## JOINT SETS AND DISJOINT SETS

- Two sets $A$ and $B$ are disjoint sets if and only if they have no common element, otherwise they are joint sets.


## Example:

$E=\{2,46,8,10\}$
$\mathrm{O}=\{1,3,5,7,11\}$
$A=\{2,3,4,5,6,7,8,9,10,11,12,13\}$

Set $E$ and set $O$ are disjoint sets since they have no common element.
Set $A$ and set $O$ are joint sets since they have some common elements. Some elements of set $O$ are also members of set $A$.
Set $E$ and set $A$ are joint sets since the elements of set $O$ are also members of set $A$.

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## SUBSETS

- Given sets $A$ and $B, A$ is a subset of $B$ if every element of $A$ is an element of $B$. In symbols, we write $A \subseteq B$ (read as " $A$ is a subset of $B$ "). We say that $A \subseteq B$ if and only if, for every $x \in A, x \in B$.


## Example:

Given:

$$
\mathrm{U}=\{\mathrm{a}, \mathrm{~b}, \mathrm{c}, \mathrm{~d}, \mathrm{e}\}
$$

$$
X=\{a, c, e\}
$$

$$
Y=\{a, c, b, e, d\}
$$

$X \subseteq Y$
$X \subseteq U$
$\mathrm{Y} \subseteq \mathrm{U}$

The elements of set $X$ are contained in set $Y$.
The elements of set $X$ are contained in set $U$.
The elements of set $Y$ are contained in set $U$.

## PROPER SUBSET

- If $A \subset B$ and $A \neq B$, we say that $A$ is a proper subset of $B$, denoted by $A \subset B$. The symbol $A \subset B$ means that every element of $A$ belongs to $B$ and $B$ contains at least one element not found in $A$.


## Example:

1.) Given: $K=\{8,12,16\}$

$$
L=\{8,12,16,20\}
$$

$$
M=\{8,12,16,20,22,23\}
$$



All elements of $K$ are contained in set $L$. L contains an element (20) not found in K . $\mathrm{K} \subseteq \mathrm{M} \quad$ All elements of K are contained in set M . M contains elements not found in K .
$L \subseteq M \quad$ All elements of $L$ are contained in set $M$. $M$ contains elements not found in $K$.

Xavier School SETS
2.) If $C=\{a, b, c\}$, name all subsets of $C$.

Solution:
\{a\}
\{b\}
\{c\}
\{a, b $\}$
\{a, c $\}$
\{b, c $\}$
\{a, b, c\} \{ \}

## NUMBER OF SUBSETS

- The number of subsets of any given set may be determined by the formula $2^{n}$, where $n$ denotes the cardinality of the number of elements of the set.


## Example

\(\left.$$
\begin{array}{|c|c|c|c|}\hline \text { Sets } & \text { Cardinality } & \text { Subsets } & \begin{array}{c}\text { Number of } \\
\text { Subsets }\end{array}
$$ <br>
\hline\{1\} \& 1 \& \},\{1\} \& 2 <br>
\hline\{0,1\} \& 2 \& \},\{0\},\{1\},\{0,1\} \& 4 <br>
\hline\{2,4,6\} \& 3 \& \{2\},\{4\},\{6\},\{2,4\},\{2,6\},\{4, \& 8 <br>
6\}, <br>

\{2,4,6\},\{ \}\end{array}\right]\)| $2^{\mathrm{n}}$ |
| :--- |

