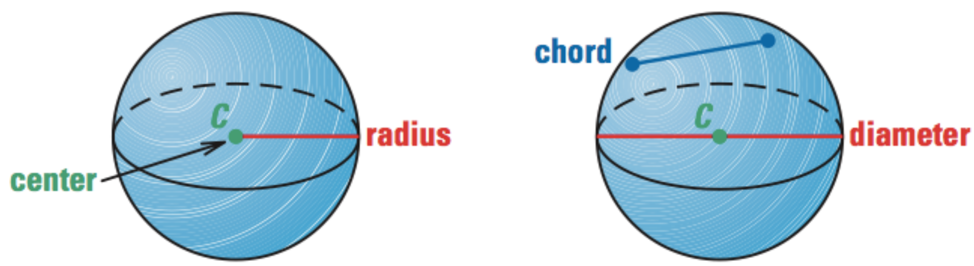
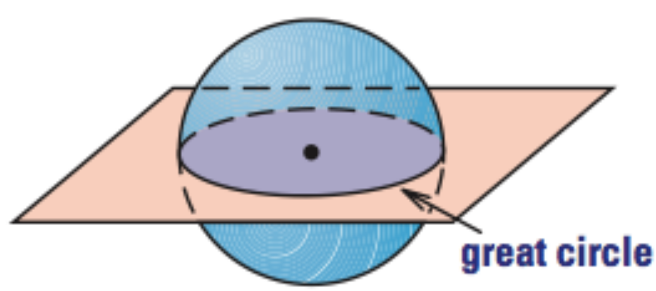
SPHERES

A sphere is the locus of points in space that are a given distance from a point. The point is called the center of the sphere. A radius of a sphere is a segment from the center to a point on the sphere.



A chord of a sphere is a segment whose endpoints are on the sphere. A diameter is a chord that contains the center. As with circles, the terms radius and diameter also represent distances, and the diameter is twice the radius.

If a plane intersects a sphere, the intersection is either a single point or a circle. If the plane contains the center of the sphere, then the intersection is a great circle of the sphere. Every great circle of a sphere separates a sphere into two congruent halves called hemispheres.

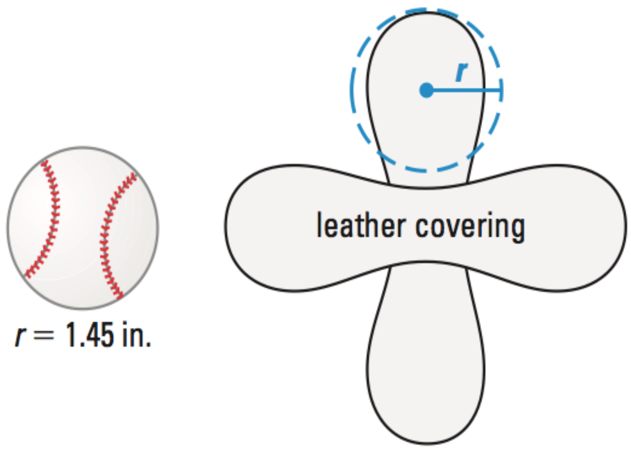


WHAT YOU'LL LEARN

- **Finding the surface area of a sphere**
- **Finding the volume of a sphere**

4 SPHERES

A baseball and its leather covering are shown. The baseball has a radius of about 1.45 inches.



a. Estimate the amount of leather used to cover the baseball.

b. The surface of a baseball is sewn from two congruent shapes, each of which resembles two joined circles. How does this relate to the formula for the surface area of a sphere?

SOLUTION:

a. the radius r is about 1.45 inches, the surface area is as follows:

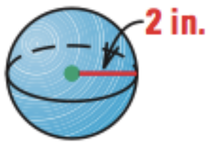
$$\begin{aligned} S &= 4\pi r^2 \\ &= 4\pi(1.45)^2 \\ &= 8.41\pi \text{ in.}^2 \end{aligned}$$

b. Since the covering has two pieces, each resembling two joined circles, then the entire covering consists of four circles with radius r. The area of a circle of radius r is $A = \pi r^2$.

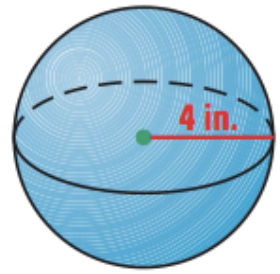
So, the area of the covering can be approximated by $4\pi r^2$. This is the same as the formula for the surface area of a sphere.

Find the surface area. When the radius doubles, does the surface area double?

a.



b.



a. $S = 4\pi r^2$

$= 4\pi(2)^2$

$= 16\pi \text{in.}^2$

b. $S = 4\pi r^2$

$= 4\pi(4)^2$

$= 64\pi \text{in.}^2$

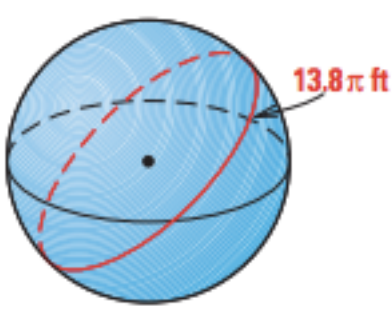
The surface area of the sphere in part (b) is four times greater than the surface area of the sphere in part (a) because $16\pi \cdot 4 = 64\pi$. So, when the radius of a sphere doubles, the surface area does not double.

SURFACE AREA OF A SPHERE

$4\pi r^2$

EXAMPLES:

1) The circumference of a great circle of a sphere is 13.8π feet. What is the surface area of the sphere?



$$C = 2\pi r$$
$$13.8\pi = 2\pi r$$
$$6.9 = r$$

$$SA = 4\pi r^2$$
$$SA = 4\pi(6.9)^2$$
$$SA = 190.44\,\pi\, ft^2$$
$$SA \approx 598.28\, ft^2$$

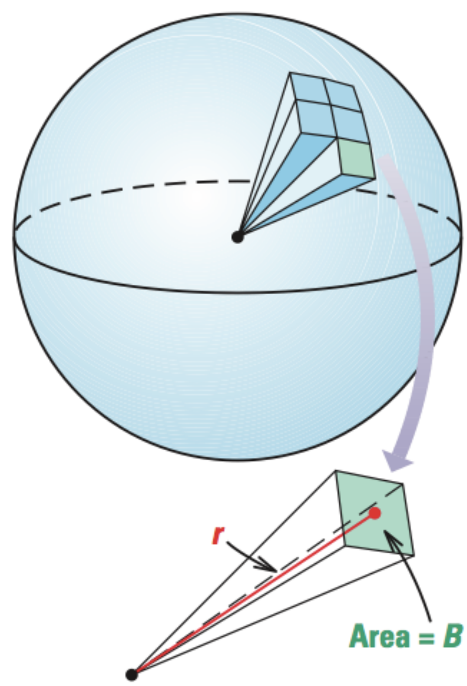
2) The circumference of a great circle of a sphere is 12 π. What is the surface area of the sphere?

$$C = 2\pi r$$
$$12\pi = 2\pi r$$
$$6 = r$$

$$SA = 4\pi r^2$$
$$SA = 4\pi(6)^2$$
$$SA = 144\,\pi\,ft^2$$
$$SA \approx 452.39\,ft^2$$

Imagine the interior of a sphere with radius r is approximated by n pyramids each with a base area of B and a height of r, as shown. The volume of each pyramid is 1/3 Br and the sum of the base areas is nB. The surface area of the sphere is approximately equal to nB, or $4\pi r^2$. So, you can approximate the volume V of the sphere as follows.

$V \approx n\frac{1}{3}Br$	**Each pyramid has a volume of $\frac{1}{3}Br$.**
$= \frac{1}{3}(nB)r$	**Regroup factors.**
$\approx \frac{1}{3}(4\pi r^2)r$	**Substitute $4\pi r^2$ for nB.**
$= \frac{4}{3}\pi r^3$	**Simplify.**

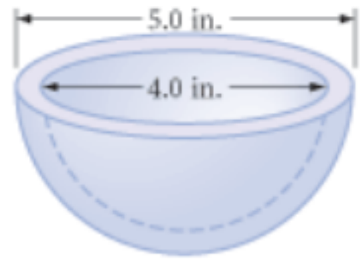


VOLUME OF A SPHERE

$$\frac{4}{3}\pi r^3$$

EXAMPLES:

a) Find the volume of plastic (to the nearest cubic inch) needed for this hollow toy component. The outer-hemisphere diameter is 5.0 in. and the inner-hemisphere diameter is 4.0 in.



Volume of outer hemisphere

Given: d = 5

$$V = \frac{\frac{4}{3}\pi(2.5)^3}{2}$$

$$V = \frac{\frac{62.5}{3}(\pi)}{2}$$

$$V = \frac{62.5(\pi)}{6}$$

$$V_{Outer} = 10.42\pi in^3$$

Volume of inner hemisphere

Given: d = 4

$$V = \frac{\frac{4}{3}\pi(2)^3}{2}$$

$$V = \frac{\frac{32}{3}(\pi)}{2}$$

$$V = \frac{32(\pi)}{6}$$

$$V_{Inner} = 5.33\pi in^3$$

Volume of Plastic needed

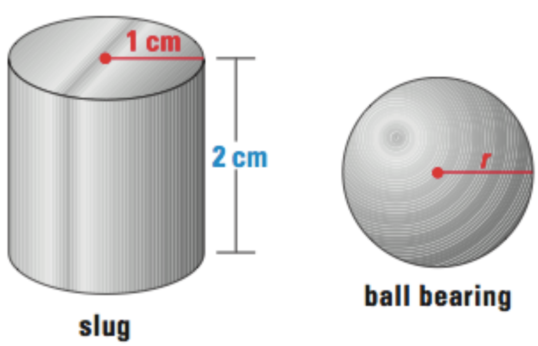
$V_{\text{Plastic needed}} = V_{\text{Outer}} - V_{\text{Inner}}$

$$V = 10.42\pi - 5.33\pi$$

$$V = 5.09\pi$$

$$V = 16in^3$$

b) To make a steel ball bearing, a cylindrical slug is heated and pressed into a spherical shape with the same volume. Find the radius of the ball bearing below.



To find the volume of the slug, use the formula for the volume of a cylinder.

$$V = \pi r^2 h$$

$$= \pi (1^2)(2)$$

$$= 2\pi\ cm^3$$

To find the radius of the ball bearing, use the formula for the volume of a sphere and solve for r.

$$V = \frac{4}{3}\pi(r)^3$$

$$2\pi = \frac{4}{3}\pi(r)^3$$

$$6\pi = 4\pi(r)^3$$

$$\frac{3}{2} = r^3$$

$$r \approx 1.14$$

So, the radius of the ball bearing is about 1.14 centimeters.

APPLICATIONS:

Solve the following problems completely. Write your final answers correct to two decimal places. Use pi in your calculator.

1) A hemisphere has a volume of 18 cm^3. Find its radius.

2) A sphere has a volume of 972 in^3. Find its radius.

3) What is the volume of the largest hemisphere that you could carve out of a wooden block whose edges measure 3 m by 7 m by 7 m?

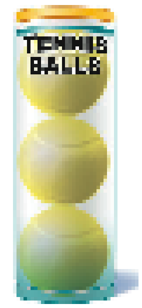
4) Find the volume of a spherical shell with an outer diameter of 8 meters and an inner diameter of 6 meters.

5) Which is greater, the volume of a hemisphere with radius 2 cm or the total volume of two cones with radius 2 cm and height 2 cm?

6) A sphere of ice cream is placed onto your ice cream cone. Both have a diameter of 8 cm. The height of your cone is 12 cm. If you push the ice cream into the cone, will all of it fit?

7) Markie's ice cream comes in a cylindrical container with an inside diameter of 6 inches and a height of 10 inches. The company claims to give the customer 25 scoops of ice cream per container, each scoop being a sphere with a 3-inch diameter. How many scoops will each container really hold?

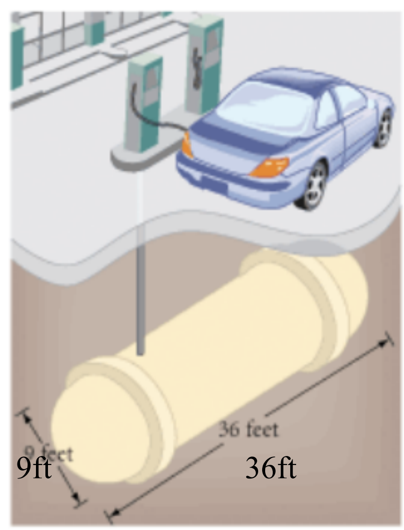
8) <u>**Packaging**</u> Tennis balls with a diameter of 2.5 in. are sold in cans of three (right). The can is a cylinder. What is the volume of the space in the can not occupied by tennis balls? Assume the balls touch the can on the sides, top, and bottom.



9) A cylindrical glass 10 cm tall and 8 cm in diameter is filled to 1 cm from the top with water. If a golf ball 4 cm in diameter is placed into the glass, will the water overflow?

10) The circumference of Earth at the equator (great circle of Earth) is 24,903 miles. The diameter of the moon is 2155 miles. Find the surface area of Earth and of the moon to the nearest hundred. How does the surface area of the moon compare to the surface area of Earth?

11) This underground gasoline storage tank is a right cylinder with a hemisphere at each end.



a. How many gallons of gasoline will the tank hold? (1 gallon = 0.13368 cubic foot.)

b. If the service station fills twenty 15-gallon tanks from the storage tank per day, how many days will it take to empty the storage tank?

References:
http://www.nos.org/secmathcour/eng/ch-24.pdf
http://www.redmond.k12.or.us/14552011718214563/lib/14552011718214563/Lesson_10.7.pdf
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