SPHERES

DETAILED SOLUTIONS

VOLUME & SURFACE AREA

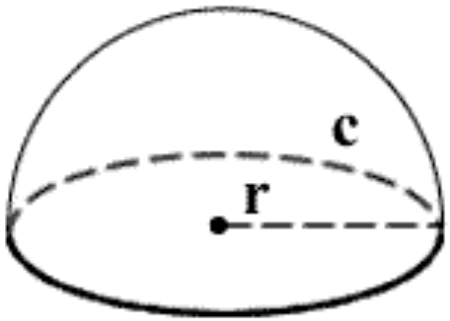
1. A hemisphere has a volume of 18 cm^3. Find its radius.

$$V = \frac{1}{2} \bullet \frac{4}{3}\pi r^3$$

$$18 = \frac{4}{6}\pi r^3$$

$$108 = 4\pi r^3$$

$$r = 2.05cm$$



or

$$2V_{hemisphere} = \frac{4}{3}\pi r^3$$

$$2(18) = \frac{4}{3}\pi r^3$$

$$36 = \frac{4}{3}\pi r^3$$

$$108 = 4\pi r^3$$

$$r = 2.05cm$$

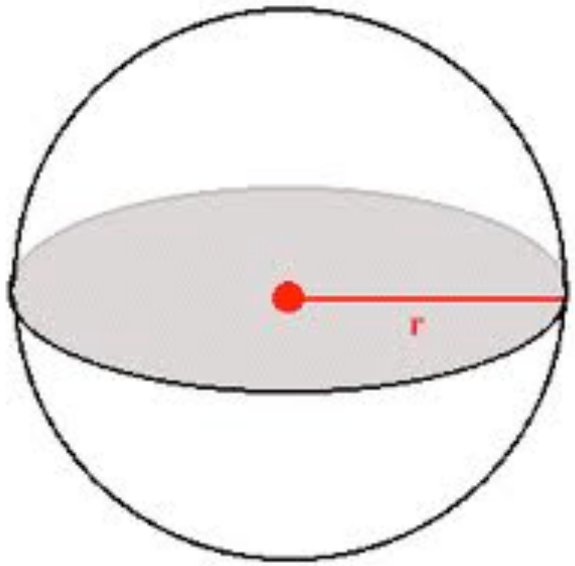
2. A sphere has a volume of 972 in^3. Find its radius.

$$V = \frac{4}{3}\pi r^3$$

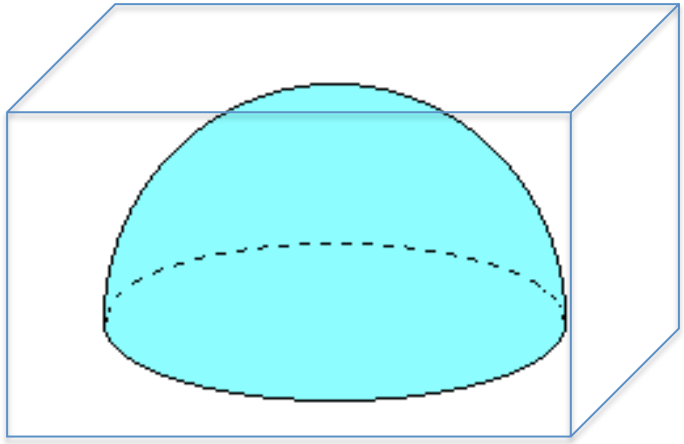
$$972 = \frac{4}{3}\pi r^3$$

$$\frac{(972)(3)}{4\pi} = r^3$$

$$r \approx 6.15\,ft$$



3. What is the volume of the largest hemisphere that you could carve out of a wooden block whose edges measure 3 m by 7 m by 7 m?



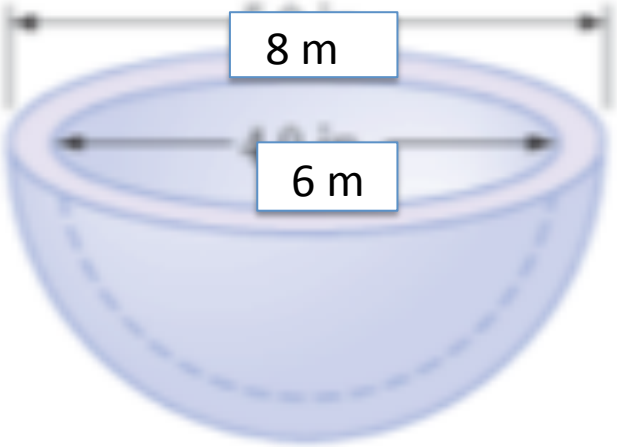
The diameter of the hemisphere could only be 3 m so it could fit inside the rectangular block.

$$\frac{1}{2} \bullet \frac{4}{3}\pi r^3 = \frac{4}{6}\pi(1.5^3)$$

$$= \frac{4}{6}\pi(3.375)$$

$$= 2.25\pi = 7.07m^3$$

4. Find the volume of a spherical shell with an outer diameter of 8 meters and an inner diameter of 6 meters.



Volume of outer – Volume of inner = Volume of shell

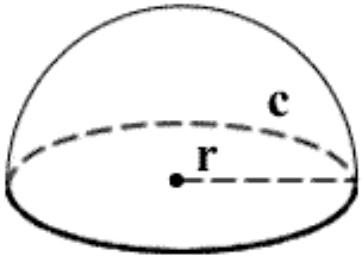
$$V = \frac{4}{3}\pi(4)^3 - \frac{4}{3}\pi(3)^3$$

$$V = 85.3\pi - 36\pi$$

$$V = 49.3\pi$$

$$V \approx 154.8m^3$$

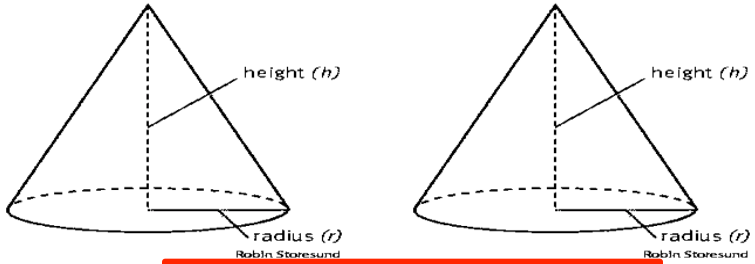
5. Which is greater, the volume of a hemisphere with radius 2 cm or the total volume of two cones with radius 2 cm and height 2 cm?



$$V = \frac{\frac{4}{3}\pi r^3}{2}$$

$$V = \frac{\frac{4}{3}(8\pi)}{2}$$

$$V \approx 16.76 cm^3$$



$$V = \frac{1}{3}\pi r^2 h$$

$$V = \frac{1}{3}\pi (2)^2 (2)$$

$$V \approx 8.38 cm^3$$

$$2V \approx 16.76 cm^3$$

The volumes are equal..

6) A sphere of ice cream is placed onto your ice cream cone. Both have a diameter of 8 cm. The height of your cone is 12 cm. If you push the ice cream into the cone, will all of it fit?

$$V_{cone} = \frac{1}{3}\pi r^2 h$$

$$V_{cone} = \frac{1}{3}\pi (4)^2 (12)$$

$$V_{cone} = 64\pi cm^3$$

$$V_{sphere} = \frac{4}{3}\pi r^3$$

$$V_{sphere} = \frac{4}{3}\pi (4)^3$$

$$V_{sphere} = 85.33\pi cm^3$$

The ice cream will not fit in the cone.

7) Markie's ice cream comes in a cylindrical container with an inside diameter of 6 inches and a height of 10 inches. The company claims to give the customer 25 scoops of ice cream per container, each scoop being a sphere with a 3-inch diameter. How many scoops will each container really hold?

$$V_{cylinder} = \pi r^2 h$$
$$= \pi(3)^2(10)$$
$$= 90\pi$$

$$V_{sphere} = \frac{4}{3}\pi r^3$$
$$= \frac{4}{3}\pi(1.5)^3$$
$$= 4.5\pi$$

each scoop of ice cream

$$\frac{90\pi}{4.5\pi} = 20 scoops$$

8. Packaging Tennis balls with a diameter of 2.5 in. are sold in cans of three. The can is a cylinder. What is the volume of the space in the can not occupied by tennis balls? Assume the balls touch the can on the sides, top, and bottom.

$$V_{tennisballs} = 3(\frac{4}{3}\pi r^3) = 4\pi(1.95)$$

$$= 24.5in^3$$

$$V_{cylinder} = \pi r^2 h = \pi(1.25)^2(7.5)$$

$$= 36.8in^3$$

Volume of the space = 36.8 – 24.5 = 12.3 in^3

9) A cylindrical glass 10 cm tall and 8 cm in diameter is filled to 1 cm from the top with water. If a golf ball 4 cm in diameter is placed into the glass, will the water overflow?

$$V_{cylinder} - V_{water} = \pi(4)^2(10) - \pi(4)^2(9)$$

$$V = 160\pi - 144\pi$$

$$V = 16\pi cm^3$$

$$V \approx 50.27 cm^3$$

$$V_{golfball} = \frac{4}{3}\pi r^3$$

$$V = \frac{4}{3}\pi(2)^3$$

$$V = 10.67\pi cm^3$$

$$V \approx 33.51 cm^3$$

The difference between the glass and the water volume is 16π or 50.27 cm³ . Since the volume of the golf ball is10.67π or 33.51 cm³, the water will not overflow because the water displaced is less than 50.27cm³.

10) The circumference of Earth at the equator (great circle of Earth) is 24,903 miles. The diameter of the moon is 2155 miles. Find the surface area of Earth and of the moon to the nearest hundred. How does the surface area of the moon compare to the surface area of Earth?

$$SA_{Earth} = 4\pi r^2$$

$$SA_{Earth} = 4\pi(3963.44)^2$$

$$SA = 62,835,426.53\pi mi^2$$

$$SA_{moon} = 4\pi r^2$$

$$SA_{moon} = 4\pi(1077.5)^2$$

$$SA = 4,644,025\pi mi^2$$

The surface area of the Earth is 13 times bigger than the surface area of the moon.